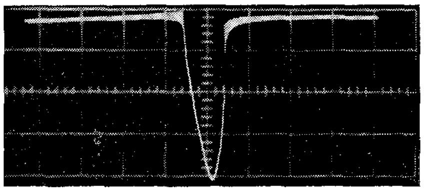

Fig. 1. Locking range of Laddertron, power injected =17 mV, power output =5 W, center frequency 34 830 Mc/s, frequency sweep =5 Mc/div, oscillator $Q=408$, [calculated from (1)].

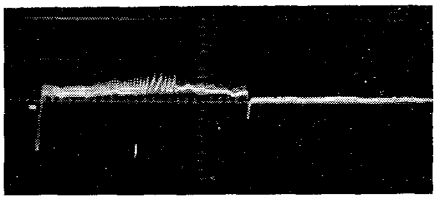

Fig. 2. Laddertron locking in pulsed mode, locked frequency =34 830 Mc/s, sweep speed $=2\times10^{-5}$ s/div, pulse width $=10^{-4}$ s, pulse power =5 W, injected power 1mW.

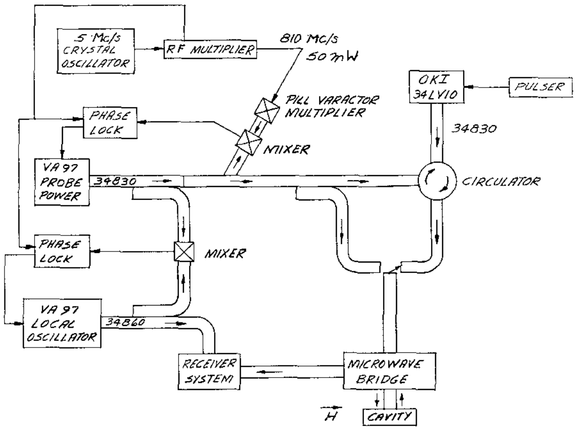

Fig. 3. Simplified block diagram of EPR spectrometer.

Q = figure of merit of the loaded oscillator cavity

ϕ = phase difference between the injected input signal and the locked output.

The locking range of the Laddertron is illustrated by Fig. 1 which shows the output of a crystal detector monitoring the reference input. Enough Laddertron output is also coupled so the mixed output is observed. The reference input is swept 40 Mc/s at 34 830 Mc/s. The video beat between the input and output is observed just before and just after locking. The lock range is 5 Mc/s. Since the response time of the oscillator is much faster than the sweep, (1) can be used to compute the loaded oscillator Q by noting the end points of the lock range which correspond to $\phi=\pm 90^0$. The dc level gives a convenient indication of the midpoint of the lock range.

Figure 2 is an oscilloscope trace of the mixed output when the reference signal was phase locked to a harmonic of a crystal oscillator and the Laddertron was pulsed on with 100-μs pulses. The injection power was reduced about 20 dB to observe the time required for locking. In the figure the Laddertron locks about 70 μs after the start of the pulse. As the reference power was increased to 17 mW, the time required for locking decreased to less than 1 μs. The synchronization time could not be accurately measured by this method.

Figure 3 is a block diagram of an electron paramagnetic resonance spectrometer. The Laddertron is used for pulse-relaxation time measurements and for the high-power multiple-quantum effects. All three tubes are locked to harmonics of the 5 Mc/s frequency standard. The probe power tube has to be phase locked by the IF offset method because the reference power from the pill varactor is low. The local oscillator is locked to the probe power by a 30-Mc/s IF phase lock. With a superheterodyne system the magnetic modulation frequency can be low (39 c/s) and by using phase-locked klystrons the IF bandwidth can be made as narrow as possible by using a crystal filter. The probe power is used to injection lock the Laddertron. The Laddertron could be locked for CW operation by the IF method but the modulation sensitivity is so low (0.2 Mc/s/V) that very high IF voltages or dc amplification of the correction signal would be required for satisfactory operation. For pulse work the injection method is required.

Improved frequency multipliers will allow the Laddertron to be locked to a crystal harmonic without the intermediate klystron.

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Nonreciprocal Directional Couplers

This correspondence is concerned with 1) the modifications required in the standard scattering-matrix description of a directional coupler [1] when the constraint of reciprocity is dropped, and 2) the advantages of using a nonreciprocal directional coupler (NRDC) in place of a conventional reciprocal directional coupler (DC) in a traveling-wave resonant circuit.

A survey of the literature reveals that those investigators who specifically studied the NRDC, such as Damon [2], Berk and Strumwasser [3], and Stinson [4], were concerned with the physics of specific models, and did not discuss the four-port characteristics of the general NRDC. On the other hand, those investigators who dealt with the more general descriptions of nonreciprocal four-ports, such as Fox [5] and Davison [6], were primarily interested in determining conditions under which a cascade of various structures would act as a circulator. More recently, Skeie [7] discussed nonreciprocal coupling of a specific type, and some characteristics of nonreciprocal four-ports but did not explore the NRDC.

To the author's knowledge the scattering matrix representation, presented below and derived elsewhere [8], of a general NRDC, (i.e., a lossless nonreciprocal matched four port with two distinct pairs of uncoupled ports) is not available in the general literature.

The standard condition for losslessness is that the scattering matrix $S=(S_{ij})$ is unitary, i.e., $S^+S=1_4$ where $(S_{ij})^+\equiv(S_{ji}^*)$ is the complex conjugate of the transpose of S, and 1_4 is the unit four-by-four matrix. For a matched four-port, we require $S_{ii}=0$ $(i=1, 2, 3, 4)$, and for the decoupled ports we have $S_{13}=S_{31}=S_{24}=S_{42}=0$. These constraints are satisfied if

$$S=\begin{pmatrix} 0 & \sqrt{1-C_1^2}\,e^{j\theta_{12}} & 0 & C_1e^{j\theta_{14}} \\ \sqrt{1-C_2^2}\,e^{j\theta_{21}} & 0 & C_2e^{j\theta_{23}} & 0 \\ 0 & C_1e^{j\theta_{32}} & 0 & \sqrt{1-C_1^2}\,e^{j\theta_{34}} \\ C_2e^{j\theta_{41}} & 0 & \sqrt{1-C_2^2}\,e^{j\theta_{43}} & 0 \end{pmatrix} \quad (1)$$

where

$$(\theta_{32}-\theta_{12})+(\theta_{14}-\theta_{34})=\pi \quad (2)$$

$$(\theta_{41}-\theta_{21})+(\theta_{23}-\theta_{43})=\pi \quad (3)$$

and where we shall refer to C_1 as the forward-coupling coefficient, and to C_2 as the backward-coupling coefficient.

For a conventional DC, the phase difference between the output waves produced by an input to any one of the four-ports [represented by the bracketed terms in (2) and (3)] are separately equal to $\pi/2$. From (2) and (3) we observe that for a NRDC, the sum of such phase differences in output waves generated by an input applied to any one port, and separately to the port isolated from that first port, are π. If (2) is sub-

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tracted from (3) we have

$$(\theta_{41}-\theta_{14})+(\theta_{23}-\theta_{32})+(\theta_{12}-\theta_{21})+(\theta_{34}-\theta_{43})=0. \quad (4)$$

In the reciprocal case the bracketed terms of (4) are separately zero.

It is interesting to observe the specific manner in which the nonreciprocal character of the device is permitted, by the constraints, to display itself. We note from (1) that the magnitude of the forward coupling between one port (say port 1) and a second port (say port 4) must equal the magnitude of the forward coupling between the port isolated from the first port (port 3) and the port isolated from the second port (port 2). See Fig. 1. A similar statement holds for the backward coupling. Thus, some degree of symmetry is preserved; lack of reciprocity manifesting itself only in the fact that the forward and backward coupling need not be equal, and $\theta_{ij}\neq\theta_{ji}$.

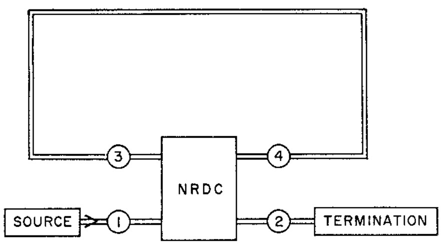

Fig. 1. Traveling-wave resonator utilizing a nonreciprocal directional coupler.

A NRDC may also be considered a generalization of a circulator, and can be usefully viewed as a means of describing an imperfect circulator.

When the primary DC in a traveling-wave resonator, TWR, is replaced by an NRDC, there are some interesting and useful consequences. For brevity we omit the detail analysis. In the simple case of a loop without reflections, the steady-state power gain for the forward waves (those traveling, say, counterclockwise in the loop of Fig. 1) is given by

$$M_f^2 = C_1^2/(1 - A\sqrt{1-C_1^2}\cos\theta + A^2(1-C_1^2))$$

where θ is the electrical length of the loop and A^2 is the relative power attenuation for a wave traversing the loop. A similar expression holds for the power gain for the backward waves in the loop, with C_2 replacing C_1. Thus, it is suggestive that the NRDC in a TWR can perform several important functions.

1) Adjusting C_1 controls the desired gain (of the forward waves).
2) Setting $C_2=0$, while C_1 has the value determined in (2), isolates the source from unwanted amplified internal reflections in the loop which without isolation are often the cause of great difficulty in tuning a ring.
3) Setting $C_2=0$, also isolates the loop from reflections from the dummy load which might be amplified in the loop.

A lengthy analysis, given elsewhere [8], of a TWR loop possessing a general source of internal reflections, and coupled by a NRDC to a source of power, verified this supposition. However, there are also some drawbacks. Unfortunately, the reflections from the dummy load would be channeled back toward the source. However, since the VSWR of these loads can be made as low as 1.1 or better, this would generally not harm the source or affect the tuning of the loop.

Another possible source of difficulty is that the internal reflection coefficient R_i is always greater for the case of $C_1\neq0$, and $C_2=0$, than for the case of reciprocal coupling $C_1=C_2=C$. On the other hand, the loaded Q for the buildup of the amplified backward waves is greater than the loaded Q for the forward waves when $C_1\neq0$, $C_2=0$. Thus, for the usual short-pulse operation of the ring the effect of the larger R_i is negated, somewhat, by the effect of the larger loaded Q on the buildup of the backward waves.

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An Optical All-Pass Network

Introduction

The problem of maintaining the correct relative phase shift between the modulation envelope components of a modulated coherent optical signal will, in all probability, eventually assume the same importance at optical frequencies that it does today at radio and microwave frequencies. Well-known phase-correcting techniques at radio frequencies are based on the application of all-pass lumped-parameter networks [1], generally constructed of inductors and capacitors in lattice and bridged-T configurations. Strip-line and transmission-line equivalents have been proposed for microwave applications [2]. Direct realization of optical all-pass filters by means of these techniques appears to be formidable. Reference [3] discusses a number of relatively unexploited microwave techniques, including the resonator-circulator principle [4] utilized in this optical all-pass device.

Optical Circulator

The circulator, illustrated schematically in Fig. 1, is similar to that proposed by S. Saito, et al. [5]. It consists of a polarizing prism beam splitter [6], a magneto-optic medium and a loss-free reflector.

The Resonator and the Reflected Wave

Consider a Fabry-Perot-like cavity of length d formed with a highly reflecting mirror on one side and a low-loss multilayer dielectric transmitting-reflecting film laid on a transparent plate on the other (Fig. 2). If a plane optical wave of unit amplitude impinges normal to the left face of the plate, then the total reflected wave, as measured at the left-hand surface of the plate is given by

$$E_R = \frac{S_{11} + (S_{11}S_{22} - S_{12}^2)e^{-i\theta}}{1 + S_{22}e^{-i\theta}}. \quad (1)$$

In (1), S_{11} and S_{22} are the (complex) reflection coefficients as seen from the left and right sides of the plate-film structure, respectively, S_{12} is the (complex) transmission coefficient, and

$$\theta = 2\frac{\omega d}{c}, \quad (2)$$

where c is the velocity of light in the intervening medium and d is the thickness of the medium (see Fig. 2). If the dielectric film is loss free, then the matrix of the scattering coefficients has the property [7]

$$[S][S]^* = [I] \quad (3)$$

where $[I]$ is the unit matrix. It follows from (3) that[1]

$$E_R = -\left(\frac{S_{12}}{S_{12}^*}\right)e^{i\theta}\left[\frac{1 + S_{22}^*e^{i\theta}}{1 + S_{22}e^{-i\theta}}\right] \quad (4)$$

or

$$E_R = \exp i\left\{(\theta + \psi_{12}) + 2\tan^{-1}\left[\frac{R\sin(\theta+\psi_{22})}{1 + R\cos(\theta+\psi_{22})}\right]\right\} \quad (5)$$

where ψ_{12} is the phase of S_{12}, ψ_{22} is the phase of S_{22}, and R is the magnitude of S_{22}. Since $|E_R|=1$ and $\arg(E_R)$ is a function of frequency, E_R has the properties of a phase dispersive all-pass filter.

Equation (6) can be put in a more tractable form by writing ω as a departure $\Delta\omega$ from center frequency ω_0,

$$\omega = \omega_0 + \Delta\omega. \quad (6)$$

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[1] It is assumed that phase is reversed at the right-hand reflector and that the phase of the scattering coefficients vary relatively slowly with frequency.